A HYDRODYNAMIC APPROACH TO COSMOLOGY:
THE MIXED DARK MATTER COSMOLOGICAL SCENARIO

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ABSTRACT

We compute the evolution of spatially flat, mixed cold and hot dark matter ("MDM") models containing both baryonic matter and two kinds of dark matter. Hydrodynamics is treated with a highly developed Eulerian hydrodynamic code [see Cen (1992)]. A standard Particle-Mesh (PM) code is also used in parallel to calculate the motion of the dark matter components. We adopt the following parameters: $h \equiv H_0/100 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1} = 0.5$, $\Omega_{cold} = 0.64$, $\Omega_{hot} = 0.3$ and $\Omega_b = 0.06$ with amplitude of the perturbation spectrum fixed by the COBE DMR measurements (Smoot $et\ al.$ 1992) being $\sigma_8 = 0.67$. Four different boxes are simulated with box sizes of $L = (64, 16, 4, 1)h^{-1}\mathrm{Mpc}$, respectively, the two small boxes providing good resolution but little valid information due to the absence of large-scale power. We use $128^3 \sim 10^{6.3}$ baryonic cells, 128^3 cold dark matter particles and 2×128^3 hot dark matter particles. In addition to the dark matter we follow separately six baryonic species (H, H^+, He, He^+, He^{++}, e^-) with allowance for both (non-equilibrium) collisional and radiative ionization in every cell. The background radiation field is also followed in detail with allowance made for both continuum and line processes, to allow non-equilibrium heating and cooling processes to be followed in detail. The mean final Zeldovich-Sunyaev y parameter is estimated to be $\bar{y} = (5.4 \pm 2.7) \times 10^{-7}$, below currently attainable observations, with a rms fluctuation of approximately $\bar{\delta y} = (6.0 \pm 3.0) \times 10^{-7}$ on arc minute scales.

The rate of galaxy formation peaks at an even later epoch ($z \sim 0.3$) than in the standard ($\Omega = 1$, $\sigma_8 = 0.67$) CDM model ($z \sim 0.5$) and, at a redshift of z=4 is nearly a factor of a hundred lower than for the CDM model with the same value of σ_8. With regard to mass function, the smallest objects are stablized against collapse by thermal energy: the mass-weighted mass spectrum has a broad peak in the vicinity of $m_b = 10^{9.5}M_\odot$ with a reasonable fit to the Schecter luminosity function if the baryon mass to blue light ratio is approximately 4.

In addition, one very large PM simulation was made in a box with size $(320h^{-1}\text{Mpc})$ containing $3 \times 200^3 = 10^{7.4}$ particles. Utilizing this simulation we find that the model yields a cluster mass function which is about a factor of 4 higher than observed but a cluster-cluster correlation length lower by a factor of 2 than what is observed but both are closer to observations than in the COBE normalized CDM model. The one dimensional pairwise velocity dispersion is $605 \pm 8\text{km/s}$ at $1h^{-1}$ separation, lower than that of the CDM model normalized to COBE, but still significantly higher than observations (Davis & Peebles 1983). A plausible velocity bias $b_v = 0.8 \pm 0.1$ on this scale will reduce but not remove the discrepancy. The velocity auto-correlation function has a coherence length of $40h^{-1}\text{Mpc}$, which is somewhat lower than the observed counterpart. In all these respects the model would be improved by decreasing the cold fraction of the dark matter and could be brought into agreement with these constraints for a somewhat smaller value of $\Omega_{CDM}/(\Omega_{CDM} + \Omega_{HDM})$. But formation of galaxies and clusters of galaxies is much later in this model than in COBE normalized CDM, perhaps too late. To improve on these constraints a larger ratio of $\Omega_{CDM}/(\Omega_{CDM} + \Omega_{HDM})$ is required than the value 0.67 adopted here. It does not seem possible to find a value for this ratio which would satisfy all tests.

Overall, the model is similar both on large and intermediate scales to the standard CDM model normalized to the same value of σ_8, but the problem with regard to late formation of galaxies is more severe in this model than in that CDM model. Adding hot dark matter significantly improves the ability of COBE normalized CDM scenario to fit existing observations, but the model is in fact not as good as the CDM model with the same σ_8 and is still probably unsatisfactory with regard to several critical tests.

Cosmology: large-scale structure of Universe – cosmology: theory – galaxies: clus-

tering – galaxies: formation – hydrodynamics

1. INTRODUCTION

In a series of papers, we have used a highly developed three dimensional hydrodynamic Eulerian code (Cen 1992) to examine the evolution of baryonic matter as well as dark matter in different model universes (standard gaussian CDM and HDM models, Cen & Ostriker 1992a(=CO92), 1992b; texture-seeded CDM and HDM models, Cen *et al.* 1991; tilted CDM model, Cen & Ostriker 1993a; PBI model, Cen, Ostriker & Peebles 1993; CDM+Λ model, Cen, Gnedin & Ostriker 1993). All were treated with the same code and the same numerical resolution. This paper is the last of this series. We study here the hydrodynamic properties of the mixed dark matter cosmological scenario which has been recently re-examined (*cf.* Davis, Summers, & Schlegel 1992=DSS herefter; Taylor & Rowan-Robinson 1992=TR hereafter; Klypin *et al.* 1993=KHPR hereafter) as a variant of the standard cold dark matter scenario. The idea for such a model dates as far back as 1984 (*cf.* KHPR for a survey of the literature), but recent observations of large-scale structure have led to renewed interest in it. It is well known that, if one normalizes the amplitude of fluctuations to the COBE DMR signal (Smoot *et al.* 1992), then the standard cold dark matter model (CDM) produces too high a small-scale velocity dispersion (Davis *et al.* 1992). There are other problems due to the shape of the power spectrum which are independent of amplitude normalization. A recent review of the triumphs and defects of the standard CDM scenario is presented in Ostriker (1993). The mixed dark matter model was proposed as an interesting alternative to the CDM model, which should produce a better agreement with observed small-scale velocity dispersion measurements and other observational constraints. The physical basis for believing in the plausibility of this approach (two species of non-interecting particles) is presented in DSS and KHPR.

The rest of the paper is organized in the following manner. Section 2 gives a brief description of the equations used [for a detailed description of equations

and numerical techniques used, see Cen (1992)]; §3 briefly describes the method to set up the initial conditions [see also Cen (1992) for a detailed description of the procedure to set up the initial conditions]; §4 gives the results of the simulations; §5 assembles our conclusions.

2. EQUATIONS AND NUMERICAL TECHNIQUES

There are two sets of equations, one for the baryonic fluid and the other for collisionless dark matter particles. For the baryonic fluid we have eight time dependent equations as follows: the mass conservation equation of total baryonic matter, the three ionization rate equations of H I, He I and He II, the three momentum equations in three directions and the energy equation. Locally, we also satisfy charge conservation and the gas equation of state: $P = n_{tot}kT$. The set of equations for the collisionless dark matter particles consists of three equations for change of momentum and three for change of position. In addition, we have the equation relating the density field to the gravitational forces, i.e., Poisson's equation for the perturbed density, and the two Einstein equations for the evolution of the cosmic comoving frame. Details of all the equations are presented in Cen (1992).

The UV/X-ray radiation field (as a function of frequency and time) is calculated in a spatially averaged fashion. Changes in other quantities are computed each time step in each cell. Ionization, heating and cooling, are computed in a detailed non-LTE fashion.

In terms of numerical technique, the dark matter evolution is computed with a standard PM code. The dark matter density and the gravitational forces exerted on dark matter particles are found using the Cloud-In-Cell ("CIC") algorithm [*cf.* Hockney & Eastwood (1981); Efstathiou *et al.* (1985)]. The gravitational potential, due to both baryons and dark matter, is calculated by solving Poisson's equation

with periodic boundary conditions utilizing an efficient FFT algorithm.

3. INITIAL CONDITIONS

We adopt the analytic fitting formulae for initial power spectrum transfer functions for both cold dark matter particles and hot dark matter particles given in KHPR. The initial power spectrum transfer function for the baryonic matter is assumed to follow that of the cold dark matter.

The normalization adopted here is $\sigma_8 = 0.67$ as in KHPR, which is fixed by the COBE DMR signals. The ratio of cold to hot matter also is taken from KHPR. Standard light element nucleosynthesis (Walker *et al.* 1991) determines Ω_b with our choice of H ($\Omega_b = 0.06$ at the upper end of the permitted range) . The initial realization of each simulation is generated by assuming that the phases of the waves are random and uncorrelated. The initial dark matter density field and baryon density field are generated using the same phase information, although the amplitude of the corresponding modes are different due to the different power spectra. The initial peculiar velocity field is then obtained by the Zeldovich approximation (*cf.* Zeldovich 1970). However, since the hot dark matter component has a non-trivial random velocity component, we try to model this velocity component for hot particles by adding in quadrature the random velocity drawn from a Fermi-Dirac distribution (following KHPR) to each pair of hot particles (with the same amplitude but opposite directions). To summarize our adopted parameters are as follows: $h = 0.5$, $\Omega_c = 0.64$, $\Omega_h = 0.30$, $\Omega_b = 0.06$ and $\sigma_8 = 0.67$, the same parameters as found to be best in DSS, RT and KHPR.

After we have made the simulations, a small error in the initial power spectrum and an error in the treatment of the initial velocity generation were brought to our attention by KHPR. But fortunately, the two errors happen to compensate one

another and the net effect is small (the rms error for position is $11h^{-1}$kpc and the rms error for velocity 1.6%).

4. RESULTS

4.1 Hydrodynamic Simulations

4.1.1 Temperature and Density

Four different models are computed with box sizes of $L = (64, 16, 4, 1)h^{-1}$Mpc, respectively. We use 128^3 cells with 128^3 cold dark matter particles and 2×128^3 hot dark matter particles in each of these simulations. Thus the nominal resolution in the four simulations ranges from $500h^{-1}$kpc in the largest box to $7.8h^{-1}$kpc in the smallest box with actual resolution in the hydro code about a factor of 2.5 worse than this. While resolution of the code is insufficient to answer many questions of interest, the *comparison* between the results found here and those presented in CO92 should be very instructive. In that paper we examined the standard CDM scenario with the normalization $\sigma_8 = 0.67$ which is the same as that adopted here.

The larger scale simulations suffer most from the defects of insufficient resolution, the smaller scale simulations from the lack of non-linear long waves which would be truly present if we had a larger box. Thus, in the largest box we know that we are underestimating cooling and condensation of self gravitating small-scale objects, whereas in the smallest box the omission of long waves makes the simulation not correct on average in that the gas will be less violently shaken and thus cooler than the average piece of the universe at that scale. Since temperatures are underestimated, the rate of condensation of self gravitating objects is overestimated in this small box (as compared to the average). The reason for this is that the Jeans mass at 10^4 K , where cooling decreases rapidly, is typically larger than the cell mass. We do not attempt to model specially those overdense regions where galaxies

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preferentially form. It is likely that our small boxes (since they have a density equal to the cosmic mean) underestimate the rate of galaxy formation in regions of high density, but they *overestimate* galaxy formation as compared to the average cosmic volume of that size. In other words, the small box is not a fair sample of the cosmic volume of that size; it would be necessary to perform many independent simulations with varying mean density (averaged over the box) to overcome this weakness to some extent.

The four simulations were run in the following order. First, we ran the $L = 16h^{-1}$Mpc box simulation, since most of the radiation which is important for ionizing hydrogen and helium comes from the scales contained in this box according to our previous tests. Second, we ran the $L = 64h^{-1}$Mpc box simulation with input radiation emissivity obtained from the $L = 16h^{-1}$Mpc run. This, we consider the simulation providing the most useful results. Third, we ran the $L = 4h^{-1}$Mpc box simulation with input radiation emissivities obtained from both the $L = 16h^{-1}$Mpc and $L = 64h^{-1}$Mpc runs. Last, we ran the $L = 1h^{-1}$Mpc box simulation with input radiation emissivities obtained from all three larger box runs. The reason we ran our models in the given order is presented in earlier papers of this series.

All the runs started at $z = 20$. As noted, the smaller boxes $L = (4, 1)h^{-1}$Mpc are useful only in a limited sense. After waves longer than the box size become nonlinear, calculations on these small scales have little validity. Besides, in this MDM model, the hot neutrino component has a thermal motion which is too large to be captured by small-scale $\leq 1h^{-1}$ potential wells even at late times; therefore, in the smaller boxes the missing long waves should have a larger effect than in the CDM model case, were they present. But these simulations provide useful information nonetheless. The large-scale (missing waves) would have heated the gas on smaller scales to higher temperatures than obtained when this long wavelength power is missing. Thus, formation of cooled, bound objects on these small scales would have

been *less* in a computation with still larger dynamic range than we have calculated in this paper. Since one of our main points (already seen in other models) is that most of the mass does not fragment into tiny lumps in the MDM picture, this point is strengthened by our inclusion of the small-scale boxes, even if they only allow an upper bound to be put on the amount of mass in isolated small-scale structures.

Figure (1) shows the actual initial (scaled to $z = 0$) and final power spectra of the four simulations. The initial baryonic power spectra are assumed to be the same as that of the cold dark matter. The vertical thick solid label shows the place in the spectrum at $8h^{-1}$Mpc which we use to parameterize the amplitude of the spectrum. We see that in the final state, at $z = 0$, on scales $\lambda > 2h^{-1}$Mpc hot dark matter component acts just like the cold dark matter component, i.e., clusterings of both components on scales $r \geq 1h^{-1}$Mpc are similar even though the initial spectra are quite different from one another. On the other hand, as expected, the hot dark matter component clusters less on smaller scales due to its thermal motion. One important point to notice is that any simulation with box size $\leq 10h^{-1}$Mpc (including the two smaller boxes in this paper) significantly underestimates the hot dark matter component density fluctuations.

Now let us turn to the results obtained. We will compare throughout with the standard CDM run we made (CO92), which has exactly the same normalization $\sigma_8 = 0.67$. On the intermediate $8h^{-1}$Mpc scale the (integrated) amplitudes are almost identical. This model has more large-scale power and less small-scale power than the CO92 run. The upper panels of Figures (2a,b,c,d) show the evolution of mean volume-weighted (solid lines), and mass-weighted (dotted lines) temperatures as a function of redshift. Heavy lines show this work and light lines show CO92 run. Also shown is the corresponding mean proper peculiar kinetic energy density (dashed lines) in units of Kelvin. The simulations are displayed in the following order: (a) $L = 64h^{-1}$Mpc, (b) $L = 16h^{-1}$Mpc, (c) $L = 4h^{-1}$Mpc, (d) $L = 1h^{-1}$Mpc

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(this order is maintained in subsequent figures.)

We see that, in the simulation with box size $L = 64h^{-1}$Mpc [Fig (2a)], the final mean mass-weighted temperature exceeds 2×10^6 K representing the small fraction of strongly shock heated gas in regions like the Coma cluster of galaxies. Similar results are found in CO92 for the standard CDM model with the same σ_8 normalization, which is expected since hot neutrinos behave essentially the same as cold dark matter on these scales. In the smallest boxes, $L = (4, 1)h^{-1}$Mpc, the mean temperatures stay at about 10^4 K because cooling processes (mainly the hydrogen and helium collisional excitation cooling) are important and the cooling time is short compared with the Hubble time, so baryonic matter can be shocked and then cool to remain at these temperatures. Besides, the shocks on these smaller scales are weaker (due to omission of waves larger than its box size, some of which would have entered the nonlinear regime at $z = 0$, were they present) compared with those in the bigger boxes, where shock heats baryonic matter to temperatures $\geq 10^6$ K .

One main difference which we found for the smaller boxes in this model compared to the CDM model is that the temperatures are much lower here, presumably due to non-clustering nature of still hot neutrinos on these scales. This difference is most noticeable in the smallest ($L = 1h^{-1}$Mpc) box [Fig (2d)].

The lower panels of Figures (2a,b,c,d) show the evolution of the variances of the baryonic and dark matter density on the scale of the cell size of each simulation, which are defined as

$$\sigma_M^2 \equiv <\rho^2>/<\rho>^2 - 1 \quad , \tag{1}$$

where $M = (d, b)$ and σ_d is the dark matter density variance, σ_b is the baryonic matter density variance. In the bigger boxes the dotted line (dark matter) shows higher fluctuations. In the smaller boxes the gas component (solid line) has higher

fluctuations. We define a bias factor as follows:

$$b(\triangle l) \equiv \frac{\sigma_b(\triangle l)}{\sigma_d(\triangle l)} \quad . \tag{2}$$

Again heavy lines are from this work and light lines from CO92. We find that on scales less than $0.125h^{-1}$Mpc, cooling processes are important, which leads to the "biased" formation of overdense baryonic objects: baryonic matter is more clumpy than dark matter on these scales. But for larger scales, cooling processes are not significant enough at later times to play an important role. Therefore, we find that on scales larger than $0.125h^{-1}$Mpc, the situation is just the opposite, i.e., dark matter is more clumpy than baryonic matter. Note that this is different from saying that the galaxy distribution follows (or does not follow) the mass distribution, since the baryonic mass distribution is significantly different from the galaxy distribution (*cf.* Katz, Hernquist & Weinberg 1992; Cen & Ostriker 1992c). Similar results were found in the study of the standard CDM model (CO92), in the tilted CDM (TCDM) model (*cf.* Cen & Ostriker 1993a) and in the CDM+λ model (*cf.* Cen, Gnedin & Ostriker 1993). As was stressed before, less cooling in the big simulation box underestimated the galaxy formation rate. But quantities such as temperature and Zel'dovich-Sunyaev effect are only affected weakly since most of the energy which eventually turns into heat is present in the simulation box; small-scale waves do not significantly contribute to the entropy generation although their effect on cooling in dense regions is, no doubt, very important.

Comparison with the standard CDM result (CO92) using the identical code (and amplitude, σ_8) is instructive. The density fluctuations in the MDM models are considerably smaller than those in the CDM models with the same box sizes, with the differences being larger for smaller boxes. This is again due to the fact that neutrinos are hot enough to escape small-scale potential wells even at $z = 0$. Quantitatively, we find that in the two bigger boxes σ_M for the gas reaches 2 [i.e.,

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$(\delta\rho/\rho)_{rms} = 2.3$] at $z = (0.6, 1.4)$ whereas it was $z = (1.0, 3.0)$ in the standard CDM run having $\sigma_8 = 0.67$. In the two smaller boxes, which give a better indication of the initiation of galaxy formation, we look for the epoch when $\sigma_M(gas) = 10$, and find $z_{10} = (0.7, 1.0)$, whereas in the CDM run this same level of nonlinearity occured much earlier at $z_{10} = (2.3, 3.4)$.

Figures (3a,b,c,d) show the volume-weighted histograms of temperatures of cells at four epochs. Figures (4a,b,c,d) show the mass-weighted histograms of temperatures of cells at the same epochs. Comparing these figures with the same numbered figure in CO92 we note the temperature distributions are similar for the largest box ($L = 64h^{-1}$Mpc) in the two cases (MDM vs CDM), while there is a trend of less hot gas in the MDM model than in the CDM model for smaller boxes. This is, once again, due to the fact that the MDM model has less small-scale power than the CDM model.

Figures (5a,b,c,d) show some typical slices with contours of baryonic matter density, total dark matter density (cold + hot components) and baryonic matter temperature at $z = (2, 0)$. Notice that linear structures are more visible in the gas than in the dark matter. These structures arise from the intersection of sheets ("pancakes") within our displayed slices. In the dark matter we expect that perturbations with $\vec{K}$ vectors within the pancakes will be relatively unstable. Note also that in Figure (5a), which shows matter on a large-scale, the dark matter is more concentrated than the baryons, whereas in Figures (5c,d), on small scales, the baryons are more clumped. A somewhat more filamentary appearance is seen in the gaseous structures in the MDM model than in the analogous CDM model, a feature expected from the work of Zel'dovich, Doroskevich and colleagues (Shandarin, Doroshkevich, & Zeldovich 1983).

In Figures (6a-6d) we contrast the structures seen in hot and cold dark matter

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particles. On the largest scale (6a) differences are not apparent to the eye, but in the smallest scale box the fluctuations in the HDM component are grossly smaller than those of the CDM component, even though both feel the same gravitational potential. This effect would clearly have the virture of providing an increasing mass to light ratio with increasing scale, a trend noticed from early investigations of the subject.

Figures (7a,b,c,d) show the (ρ, T) contour plots. The innermost contours represent conditions of density and temperature in the most common cells at $z = 0$. Comparison with standard CDM model is very informative. Note that in all three smaller boxes there is a distinctive feature, i.e., certain regions with high densities $(\rho/\bar{\rho} > 10^2)$ but relatively low temperatures ($\sim 10^4$ K) where the gas is near the peak of the cooling curve. We found the same feature in standard CDM model (CO92) but the largest densities (with low tempeatures) are smaller in this MDM case than in the CDM case. Also, there are regions having both high densities and high temperatures in Figure (7a) representing the hot X-ray emitting gas in the great clusters. On all scales the most common cells have a density $\rho/\bar{\rho} \simeq 0.1$ and are in "voids".

4.1.2 Volume and Mass Distributions

We now analyze the simulations in another quantitative way. The baryonic matter in each simulation box is divided into four components: (1) virialized, bound, hot objects, which on the large-scales represent the gas in clusters of galaxies and on the small scales represent the L_{α} clouds — "Virialized Gas"; (2) bound, cooled objects, i.e., collapsed compact objects — "Galaxies"; (3) unbound, hot regions with temperature $\geq 10^5$ K — "Hot IGM "; (4) other regions, primarily — "Voids". The break point at 10^5 K is adopted because it is past the peak of the "cooling curve". The quantitative definitions of these regions are given in CO92.

These four components make a complete set of possible objects and each cell is classified accordingly. In Tables (1 - 3) we list the volume and mass weighted fraction of these four components at six epochs, $z = (10, 5, 3, 2, 1, 0)$, for the three different runs with box sizes, $L = 64, 16, 4h^{-1}$Mpc. In the preceding tables we do not treat "galaxy formation" as irreversible, which it would be were a true stellar component to be formed. Thus, fewer cells satisfy our criterion to be "galaxies" after $z = 1$ than at that epoch for Table 1. The result for the $L = 1h^{-1}$Mpc is not tabulated because we find that all the cells belong to "Voids".

In the $L = 64h^{-1}$Mpc box, in comparison with the CDM simulation, a larger fraction of the mass is in voids, and "galaxy formation" is slightly later and less vigorous. And in all the boxes galaxy formation is later in MDM model than in the standard CDM model. This comparison is also shown in Figure (8). The most important difference is that galaxy formation is later in the MDM model, especially at earlier times, than in the CDM model with the same σ_8. For example, at $z = 3$, the galaxy formation rate is, by an order of magnitude, lower in the MDM model than in the CDM model, worsening the already difficult situation of high redshift quasar formation in the (biased) CDM model. KHPR, addressing this problem in the context of the approximate Press-Schechter formalism, conclude that the model is marginally consistent with the observed existence of high redshift quasars. Haehnelt (1993), examining a broader observational data base and taking a somewhat more critical attitude, argues that the model considered in this paper ($\Omega_H = 0.3$) (and by the other quoted authors) is *inconsistent* with observations of high redshift quasars.

Now let us look at the properties of the typical collapsed objects of the four boxes at redshift $z = 1$. The results are shown in Table (4). We note that the mass weighted mass function (i.e., mass fraction of collapsed objects) has a peak around $<m> = 10^{9.3} M_\odot$. But we think that this peak would be shifted to a still larger mass

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scale were long missing waves in the smaller boxes included as they would have heated up the gas medium and stablized instabilities on small scales. In a better calculation, with all the longer waves included, the collapsed fraction would clearly peak at a still larger mass scale than shown in Table (4), since the temperature and hence Jeans mass would be higher. In addition, extra energy input from star formation, were it included, would also increase the temperature and further stablize small-scale perturbations. On all scales the largest fraction of the mass is in the IGM with about 2/3 in the "Voids" ($T < 10^5$K) and about 1/3 in the "Hot IGM" ($T \geq 10^5$K). A principle difference between MDM model and standard CDM model is that in CDM model (CO92) we found a slightly larger mass fraction in the hot ("Hot IGM") component than that of "Voids". Here most of the baryonic mass (as well, of course, as most of the volume) is in the voids. In addition, and more significantly, the galaxy fraction is much less in the MDM picture as can be seen by comparing Table 5 and with the same numbered table of CO92.

4.1.3 X-ray Background Radiation

We have calculated the mean UV/X-ray background radiation field as a function of frequency as well as time including absorption by hydrogen and helium and both free-free and free-bound emission processes. Figure (9) shows the results at six epochs, $z = 5$ (solid line), $z = 3$ (dotted line), $z = 2$ (short, dashed line), $z = 1.5$ (long, dashed line) $z = 0.5$ (dotted, short-dashed line) and $z = 0$ (dotted, long-dashed line). Emissivities from both $L = 64h^{-1}$Mpc and $L = 16h^{-1}$Mpc runs are included. The box in the middle shows the observational data by Wu *et al.* (1991). We see that the computed MDM model fails by a factor of 50 to produce the observed soft X-ray (0.2 to 1Kev range) background. The deficit at harder X-rays (1 to 10 KeV range) is even larger (note that at the high frequency end the computed spectrum has a very steep slope). There are two correction terms which

need to be taken into account. First, much of the background is in fact produced by identifiable AGN sources. We assume here that approximately half of the X-ray background radiation is due to discrete AGN sources. Second, for purely numerical reasons, we know that with the same input parameters, a better treatment with larger N, larger L_{max} and smaller L_{min}, would increase the X-ray output. A factor of 3 increase was found at $1KeV$ in the tests we made in Cen (1992) for a 128^3 CDM run with $L = 64h^{-1}$Mpc, $h = 0.5$, $b = 1.0$, $\Omega = 1$ and $\Omega_b = 0.1$. Combining these two factors indicates that this MDM can make a small but non-trivial 12% of the residual soft X-ray background radiation field, approximately 1/4 the fraction of the CDM model having the same value of σ_8. This is an improvement over the COBE normalized standard CDM model which overproduces both the X-ray background and correspondingly the number of high luminosity X-ray clusters (Frenk *et al.* 1990; Kang *et al.* 1994; Bryan *et al.* 1994).

The strong edges seen in the spectra at 13.6 eV are due to absorption by neutral hydrogen. Meantime, the edges at the 54.4eV absorption edge due to once-ionized helium is less significant simply because there is much less of this species. The edge at the ionization potential of neutral helium, 24.6eV, is seen at early epochs, but is smaller because the 24.6eV edge is too close to the L_α 13.6eV edge to be very noticable at our displayed resolution given the redshift smearing. At $z = 0$, hydrogen and helium are still not completely ionized, the troughs all remain. Again one should be reminded that energy feedback (e.g., UV and supernova processes) from star formation was not included in the simulations; the effects of these processes will be smaller than in the CDM model having the same value of σ_8.

We have computed, but not shown, figures for the ionization state and opacity [Gunn-Peterson (1965) effect] for this model. Needless to say, without UV from star formation and supenova energy input into the IGM from young galaxies, the MDM model is far from satisfying the Gunn-Peterson test of the high redshift

quasar observations, i.e., the IGM cannot be ionized by means of shock heating, bremsstrahlung and free-bound radiation. However, given the nature of late galaxy formation seen in the MDM model (Fig. 8), we think that radiation from star formation is not likely to eliminate this discrepancy. The reasoning is comparative: $b = 1.3$, CDM with UV input from galaxies is barely satisfactory (Cen & Ostriker 1993b) at $z = 4$ (*cf.* Figure 8 of the above referenced paper) and galaxy formation is lower by approximately 10^2 in the MDM model. At $z = 5$ we find that less than 10^{-5} of the baryons will have collapsed to possibly form galaxies. High mass stars with a normal mass function burn $< 10^{-2}$ of the mass with an efficiency of $10^{-2.5}$ into ionizing photons. Propagating these parameters through the ionization equations Miralda-Escude & Ostriker (1990) conclude that a collapsed fraction of 10^{-4} was marginally satisfactory to satisfy observed Gunn-Peterson limits and that 10^{-5} would marginally fail. Tagmark & Silk (1993) come to a similar conclusion concerning very late ionization in the MDM scenario.

4.1.4 Zeldovich-Sunyaev Effect

Now we turn to the results of the directly computed mean Zeldovich-Sunyaev y parameter at six epochs, $z = (5, 3, 2, 1, 0.5, 0)$ shown in Table (5). Note that $\sim 95\%$ (90% in standard CDM model) of the contribution to the Zeldovich-Sunyaev effect comes from the epochs between $z = 1$ to $z = 0$. Also the final ($z = 0$) y parameter has a lower value in MDM model than in CDM model. The reason for this difference is due to the fact that small scales waves ($\lambda < 16h^{-1}Mpc$, which contribute most to entropy generation) enter nonlinear regime later in the MDM model than in the CDM model. Let us emphasize that Table (5) shows the *directly* computed value of the y parameter. Due to our inevitable numerical inadequacies we have underestimated $\bar{y}$. Using the extrapolation formula derived in Cen (1992) (*cf.* equation (76) of that paper), if we had included all the waves and had infinite

resolution in the calculation, we would have obtained the following extrapolated value and an estimated fluctuation of y at $z = 0$ when $(N^{-1}, L_{max}^{-1}, L_{min}) \rightarrow 0$, $\bar{y} = (5.4 \pm 2.7) \times 10^{-7}$, $\bar{\delta y} = (6.0 \pm 3.0) \times 10^{-7}$ on arc minute scales (where the $\pm$ indicates our estimate of the error of our extrapolation procedure). If the reader distrusts our extrapolation procedure, then Table (5) can be taken as a firm lower bound on $\bar{y}$ for the adopted model.

4.1.5 Galaxy and Dark Matter Correlation Functions

A cell belonging to the second category defined in §4.1.2 is called a galaxy. Further, such cells, if adjacent, are grouped into a single "isolated galaxy", although at our resolution we can not tell the difference between galaxies and small groups such as the Local Group. We have found 1502 such "isolated galaxies" at $z = 1$ in the $L = 64h^{-1}$Mpc box. The reason we identify the galaxies at $z = 1$ instead of $z = 0$ is for the convenience of comparison with equivalent CDM simulation, where galaxy formation strongly peaks at $z = 1$, since the breaking of long waves at later times heats up the baryonic matter causing evaporation of earlier identified galaxies. In the present model there is still a similar mass fraction of galaxies at $z = 0$ compared to $z = 1$ [i.e. less "evaporation" *cf.* Figure (7)] due to weaker shocking in this model. In a more realistic calculation the transition to collisionless (stellar) material would be irreversible. We also randomly selected 2900 dark matter particles over the whole box ($L = 64h^{-1}$Mpc box) at $z = 1$, which is a good approximation for the representation of the total mass distribution.

Figure (10) shows the galaxy-galaxy (open circles) as well as cold dark matter particle-particle (solid dots) two-point correlation functions in the simulation with box size $L = 64h^{-1}$Mpc at $z = 1$. The errorbars are one sigma Poisson fluctuations. Also shown is $\xi(R) = (R/5h^{-1}\text{Mpc})^{-1.8}$ (dotted line), the observational data for galaxies (*cf.* Davis & Peebles 1983) scaled down by a factor of $1/2^2$. This factor

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is the linear growth factor from $z = 1$ to $z = 0$ in this model. It shows that the galaxy distribution is strongly biased over the mass distribution at this epoch with the bias factor of about 4. The slope of the galaxy-galaxy correlation is roughly consistent with observed value (-1.8) at $z = 0$. Those results are also consistent with those of KHPR in that the bias needed was found to be approximately 1.9. The apparent bias shown by the distance between the open and filled circles (square root thereof) is too large by about a factor of 1.7 but is not trustworthy. A more precise comparison with observations awaits a detailed treatment of this scenario (similar to that given by us for the standard CDM scenarion, Cen & Ostriker 1993c), where the galaxy subunits are produced irreversibly and followed with the PM code. The reason for a significantly stronger bias in the MDM model than in the CDM model is that only fairly deep potential wells are capable of collecting hot neutrinos causing deepening of the potential wells and hence inducing galaxy formation. But the bias is likely to be weaker at $z = 0$, when the neutrinos are cooler.

4.1.6 Mass Functions and Multiplicity Functions

A cell is called a bound cell if it satisfies the following criteria:

$$\phi + 0.5 * (v^2 + C^2) < -0.5v_b^2 \quad , \tag{3}$$

where ϕ is the proper peculiar gravitational potential; v is the proper peculiar velocity; C is the local speed of sound. We take, as in early papers of this series, (somewhat arbitrarily) $v_b^2 \equiv 141^2 (\mathrm{km/s})^2$; here v_b^2 is the binding energy per unit mass. We choose such a value of v_b to satisfy the requirement that about 70% of the galaxies are in groups/clusters as observations indicate [Gott & Turner 1977, Figure (2)]. The definition we have used is arbitrary but corresponds roughly to what observers identify as "bound groups". Of course in an $\Omega = 1$ universe *all* galaxies are bound to all other galaxies. After we have found these bound cells, we

group them into a number of "groups" within each group all the cells are connected (i.e., touching by at least one side of a cell). The multiplicity function of the bound groups, which are defined above is shown as solid histogram in Figure (11); also shown (dotted) is the same function for the $b = 1.5$ standard CDM model as dashed histogram. The difference is statistically significant for small groups but we do not know which is in better accord with modern data.

Figure (12) shows the baryonic and total mass/multiplicity functions of collapsed objects. Open triangles, filled dots and filled triangles are collapsed objects from three different boxes, with box sizes $L = (64, 16, 4)h^{-1}$Mpc, respectively, at $z = 1$. As noted earlier, the results in the smaller boxes overestimate the amount of bound material. The upper panel shows the baryonic mass/multiplicity function of collapsed objects, and the dashed line is a fitting formula [equation (5), see below]. The lower panel shows the total mass/multiplicity function of collapsed objects, and the dashed line is a fitting formula [equation (6), see below].

$$f(M_{bar})dM_{bar} = 0.06 * (M_{bar}/M^*_{bar})^{-1.3}e^{-M_{bar}/M^*_{bar}}d(\frac{M_{bar}}{M^*_{bar}}) \quad , \qquad (4)$$

where M_{bar} is the baryonic mass in units of solar mass, $M^*_{bar} = 1.5 \times 10^{11} M_\odot$.

$$f(M_{tot})dM_{tot} = 0.01 * (M_{tot}/M^*_{tot})^{-1.3}e^{-M_{tot}/M^*_{tot}}d(\frac{M_{tot}}{M^*_{tot}}) \quad , \qquad (5)$$

where $M^*_{tot} = 5 \times 10^{12} M_\odot$.

Taking the ratio of the fitted number density of simulated galaxies at M^*_{bar} to the observed number density of galaxies at L^* (Schechter 1976) gives an estimate of the baryonic mass to blue light ratio, $(M/L)_1 = 1.5$. We obtain a second baryonic mass to light ratio by matching the fiducial luminosity of $L^*_{B(0)} = 1.3 \times 10^{10} L_\odot$ with M^*_{bar}, which ratio found to be $(M/L)_2 = 1.5 \times 10^{11}/1.3 \times 10^{10} = 11.5$. The second estimate is somewhat higher than the first one, in part due to the low resolution of our simulations. For example, we are not able to resolve a system like the Local

Group into separate galaxies. It is interesting that these estimates are not grossly inconsistent with one another and both are not far from what is obtained in the Galactic disc via the Oort limit or in globular cluster with the virial theorem. If we take the geometric average of these two estimates, $(M/L) = 4.2$, inserting this derived mass-to-light ratio into Schechter's original formula yields:

$$\phi(M)dM = 0.04(\frac{M}{M^*})^{-1.24}e^{-M/M^*}d(\frac{M}{M^*}) \quad , \tag{6}$$

where $M^* = 5.4 \times 10^{10} M_\odot$. This is shown as the solid line in upper panel of Figure (12). In so far as the solid line fits the computations, we can say that the derived mass functions for collapsed baryonic matter are consistent with observations when a baryonic mass to light ratio of $4 - 5$ is adopted. The dashed line in the lower panel corresponds to a Schechter fit with $M_{tot}/L_B = 380$ similar to the value found observationally in clusters of galaxies (Trimble 1987).

4.2 A Very Large PM Simulation

In order to study statistical properties of clusters of galaxies as well as those of galaxies on large-scales, a larger simulation volume is desired. Our hydrodynamic simulations, although providing much more detailed physical treatment, do not have a large enough volume for this purpose. Besides, a collisionless PM approach should be valid on very large-scales where thermodynamic processes play a much less important role than on smaller scales. We made one large PM simulation with a box size of $320h^{-1}$Mpc, $200^3 = 10^{6.9}$ cold dark matter particles $2 \times 200^3 = 10^{7.2}$ hot dark matter particles utilizing a 400^3 mesh. The resolution of this simulation is $0.8h^{-1}$Mpc, which is adequate for the study of masses and correlation functions of rich clusters. The volume ($14h^{-3}$Mpc^3) within the Abell radius ($1.5h^{-1}$Mpc) corresponds to about 28 cells. This 640Mpc simulation is to be compared with the 200Mpc simulation by KHPR and the 14Mpc simulation by DSS. Our resolution

with a 400 mesh is nominally 1.6Mpc, the same as the nominal resolution of KHPR and far larger (worse) than the resolution of the small-scale P^{3}M simulation of DSS. Thus our work should (numerically) provide the best statistical information about large scale features (bulk flow, cluster-cluster correlations etc) and DSS the best information about the small-scale dark matter distribution.

4.2.1 Power Spectrum

Figure (13) shows the initial (linearly scaled to $z = 0$) final power spectra of the PM simulation. The thick, solid line is the initial ($z = 20$) power spectrum of the cold dark matter component in the MDM model. The thick, dashed line is the initial power spectrum of the hot dark matter component. The thin, solid line is the final ($z = 0$) power spectrum of the cold dark matter component. The thin, dashed line is the final power spectrum of the hot dark matter component. For the purpose of comparison also shown is the final (z=0) power spectrum for the COBE-normalized CDM model (thin dotted line). We see three things in this figure. First, the initially ($z = 20$) noticeable difference in the HDM and CDM power spectra on scales ($\lambda < 5h^{-1}Mpc$) diminishes at $z = 0$ (for $\lambda > 2h^{-1}$Mpc) due in part to the nonlinear evolution and in part to the interactions (gravitationally) between CDM and HDM components. Second, the final power spectra (both MDM and CDM models) have a slope of ~ -1 in the range $\lambda = 5 - 30h^{-1}$Mpc; this is a purely nonlinear effect. Finally, the COBE-normalized CDM model has much higher fluctuations on scales $1 - 80h^{-1}$Mpc than has the CDM model, the largest difference being 2.3 in amplitude on the scale $\lambda \sim 6h^{-1}$Mpc.

4.2.2 Correlation Function

Figure (14) shows the two-point correlation functions for cold dark matter particles and hot dark matter particles separately. In the left hand panel at $z = 0$ we see that, on scales $\geq 1h^{-1}$Mpc, cold dark matter and hot dark matter are

distributed similarly. In other words, the initially hot neutrinos have cooled down sufficiently by $z = 0$ that they have fallen into the gravitational potential wells of cold dark matter. The right hand panel shows the situation at $z = 2$. We see that at that epoch a small difference between the two species remained.

4.2.3 Cluster Properties

Now we turn to the clusters of galaxies, which are the largest known gravitationally bound systems in the universe. We here concentrate on three fundamental observables for clusters of galaxies: the cluster-cluster two point correlation function, the cluster mass function and the cluster merging rate. For this set of issues our PM simulation should have significant advantage over prior work on the MDM scenario.

We select the clusters using an adaptive friends-of-friends linking algorithm. Then we determine the linking length b_{ij} between the i-th and j-th particles by $b_{ij} = \text{Min}[L_{\text{box}}/N^{1/3}, \beta(\frac{1}{2})^{1/3}(1/n_i(a_s) + 1/n_j(a_s))^{1/3}]$, where L_{box} is the box size, N is the total number of particles in the box, $n_i(a_s)$ is the local number density at the i-th particle's position smoothed over a gaussian window of a_s. We use $a_s = 10h^{-1}\text{Mpc}$ and $\beta = 0.25$. The linking scheme is not sensitive to a_s (e.g., $a_s = 5$ or $10h^{-1}\text{Mpc}$ yields similar results). The β parameter was selected by testing that the linked groups are neither considerably smaller than the typical $1.5h^{-1}\text{Mpc}$ radius observed for rich clusters (see below) as would happen for small β, where only the small dense cluster core is linked, nor considerably larger as would occur for too large a β, where clusters are linked with other neighboring clusters. The results are not sensitive to small variations in the selected β (Bahcall & Cen 1992). From this catalog of grouped objects we select all clusters above a threshold mass within a sphere of $1.5h^{-1}\text{Mpc}$ radius of the cluster center (for proper comparison with observations). This yields a final list of clusters and their $1.5h^{-1}\text{Mpc}$ masses

at $z = 0$.

Figure (15) shows the computed cluster mass functions for MDM model as well as that from observations (Bahcall & Cen 1993). We see that this model predicts a cluster mass function about 4 times higher than observed (with observed masses determined by virial and X-ray temperature methods agree well with one another, Lubin & Bahcall 1993), which is significant since the observational uncertainty is about a factor of 2. KHPR agree in their estimate of the mass function predicted by the model but stress the observational uncertainties. While the observations are certainly incomplete we doubt that this can account for the discrepancy. The error is of the same sign but not as large in amplitude as for the pure CDM model. A pure HDM model, normalized to COBE, of course produces too few clusters, thus we expect that one could find an MDM model with $\Omega_{CDM} < 0.7$ which would be satisfactory with regard to this test. However, such a model would be worse with regard to the early formation of structure.

Figure (16) shows the two point correlation function of Abell $R \geq 1$ clusters in real space, with mean separation of $55h^{-1}$Mpc, from our simulated Abell clusters and from observations (Bahcall 1988) (the computed correlations on scales $r \leq 5h^{-1}$Mpc is probably underestimated due to our limited numerical resolution of the cluster identification scheme). We see that the cluster correlation in this model is marginally consistent with observations and is better than the COBE normalized CDM model and significantly uncertainty still exists concerning the observational situation..

Figure (17) shows the merging rate in the MDM models and also the CDM model ($\sigma_8 = 0.77$) for comparison. The measure of merging in Figure 17 is based on an identification of cluster-like gravitating systems described above. By comparing the member particles of each cluster at $z = 0$ with the member particles of each

cluster at redshift z, we identify the parent cluster for each present-day cluster as the cluster at redshift z with the maximum number of overlapping members. Then the fractional mass change in the cluster is

$$\left(\frac{\Delta M}{M}\right)_z = 1 - \frac{M_z}{M_0},\tag{7}$$

for parent and present cluster masses M_z and M_0. We compute this statistic for the most massive clusters in the simulation, with the lower mass limit chosen so the comoving number density is $9.4 \times 10^{-6} h^3 \mathrm{Mpc}^{-3}$. Although the normalizations for the two models and the final distribution of dark matter are similar, there is a much larger cluster merging rate in the MDM model than in the CDM model. We believe that the reason is that at later times when neutrinos get sufficiently cooled down, they start to be collected at the great clusters. At early times the potential wells are less deep and the neutrinos are hotter; the two effects cooperate to reduce the effective Ω for cluster material.

In MDM the median change in the cluster mass is $(\Delta M/M)_{0.3} = 0.52$ from $z = 0.3$ to the present, and $(\Delta M/M)_{1.0} = 0.90$ from $z = 1$. These are considerably larger than the corresponding values $(\Delta M/M)_{0.3,1.0} = 0.29$ and 0.77 for CDM. The rapid merging rate in CDM is discussed by Frenk *et al.* (1990). We suspect that the evolution of the great cluster properties will be different enough between MDM and CDM (*cf.* also Figure (15) to provide a meaningful comparative test.

4.2.4 Velocity Information

We compute two statistics with regard to the velocity field. First, in Figure (18a) we show the one-dimensional relative velocity dispersion defined as

$$v_{1d} = \langle [v_x(1) - v_x(2)]^2 \rangle^{1/2}/\sqrt{3}.\tag{8}$$

This is averaged over particles, that is, v_{1d} is a mass-weighted statistic. At $1h^{-1}\mathrm{Mpc}$ separation the rms value for the 1d velocity dispersion is $605 \pm 8\mathrm{km/s}$. Correcting

this for the velocity bias that we found on the $1h^{-1}$Mpc scale in Cen & Ostriker (1992c) of 0.8 ± 0.1 (for the very similar $b = 1.3$ CDM model) we find $v_{1d}(gal) = 484 \pm 6$ which is to be compared with 340 ± 40km/s. The discrepancy remains but is considerably less than in the COBE normalized standard CDM model. Also shown is the data from Davis & Peebles (1983). It is seen that this MDM fares similarly as the standard CDM model with same σ_8, but in disagreement with observed value. The physical velocity bias (*cf.* Cen & Ostriker 1992c) of $b_v = 0.8$ we see ($1h^{-1}$Mpc) is not able to bridge the gap. We have compared our results with these of KHPR with regard to this all important statistic. Specifically, Figure (18) can be compared with Figure (10) of that paper. Qualitatively the two sets of results show a similar dependence on r in the range 2Mpc$< rh < $8Mpc, where both calculations might be valid. But despite the identical assumed power spectra and normalizations, and very similar numerical methods, our results for v_{1d} are larger than those in KHPR by about a factor of 1.5. The difference is large enough so that KHPR could assert satisfactory agreement with observations, whereas we find that the disagreement is probably significant.

What is the truth here? We believe that the difference is primarily due to our larger box size ($320h^{-1}$Mpc in our case vs $25h^{-1}$Mpc in KHPR), which allows longer waves and more high velocity dispersion clusters, and due to the fact that in KHPR pairs with velocity difference greater than 1000km/s are excluded. We did the following exercise to test this hypothesis. We randomly select 100 boxes of size $25h^{-1}$Mpc within our $320h^{-1}$Mpc simulation box and computed the above statistic separately for each of the subboxes. We then group the results to show the dependence on the mean density of the subbox being studied. The results for the 1-d velocity dispersion at $1h^{-1}$Mpc separation as a function of mass overdensity of the subboxes relative to the global mean are shown (Figure [18b]) for two case: the open circles with and filled dots without velocity pairs > 1000km/s. Also shown are the

value from KHPR (thin horizontal arrow at the left axis) and our computed value (thick horizontal arrow). The dashed histogram indicates the distribution (shown by the right vertical axis) of the subboxes as a function of their overdensities. Note that the average pairwise velocity dispersion (the open circles weighted by the dashed histogram) is not the same as indicated by the thick solid arrow, since the former is a volume-sampling and the latter is a particle-sampling.

By construction, the mean density of the KHPR $25h^{-1}$Mpc box was unity, and their result (thin arrow) is consistent with what we obtained from our subset of boxes with $0.8 < \rho/\langle\rho\rangle < 1.2$. We see that it is not surprising that KHPR obtained a lower value (by a factor of 1.5) than ours. The larger value found in our work is 605 ± 69km/s (1σ dispersion) or ± 8km/s (probable error) due simply to use of a larger box which can include more long wavelength power.

Next, in Figure (19) we show the scalar correlation function for the mass peculiar velocity field defined as

$$\psi(r) = \text{sign}|\langle \vec{v}(\vec{x}) \cdot \vec{v}(\vec{x} + \vec{r})\rangle|^{1/2}, \tag{9}$$

again mass weighted. The prefactor means ψ is given the sign of the autocorrelation function. We see that the coherence length l_v (defined as the scale where this statistic drops to the value half that at zero separation) is $\sim 40h^{-1}$Mpc in agreement with that of the standard CDM model, but smaller than some recent observations which indicate very large-scale bulk motion (Lauer & Postman 1992).

4.2.5 Dipole Issue

We consider finally the relation between the large-scale mass distribution and the peculiar velocity of the Local Group. In linear perturbation theory, the peculiar velocity at position $\vec{r}$ produced by the mass distribution represented by point masses m_i at positions $\vec{r_i}$ is

$$\vec{v} = \frac{GH_o\Omega^{0.6}}{4\pi G\rho_b} \sum m_i \frac{\vec{r_i} - \vec{r}}{|\vec{r_i} - \vec{r}|^3}. \tag{10}$$

The scaling with the density parameter Ω is a useful approximation if the cosmological constant vanishes or if the universe is cosmologically flat (Peebles 1984). In an application of equation (10) to a catalog of mass markers, the sum must be truncated at some maximum distance R. The truncation causes a misalignment of the predicted velocity and the observed velocity $\vec{v}_{lg}$ of the Local Group relative to the CBR, and, if the observed and predicted values of v_{lg} are used to estimate Ω, the missed mass fluctuations beyond the depth of the catalog can produce a systematic overestimate of the density parameter (Juszkiewicz, Vittorio, & Wyse 1990). We investigated these effects in the MDM model runs by comparing the prediction of equation (10) when the sum is truncated at distance R (by a Gaussian window $e^{-r^2/2R^2}$) to the actual peculiar velocity computed as the weighted sum

$$\vec{v} \equiv \sum \vec{v}_i W_i / \sum W_i, \tag{11}$$

where the $\vec{v}_i$ are the dark matter particle velocities and the weight function decreases linearly with distance from the chosen origin to $W_i = 0$ at distance $r = 2.5h^{-1}\mathrm{Mpc}$. Equation (11) averages over the small-scale motions, as one does for the motion of the Local Group, while preserving the velocity field the scales where we can trust our code. Figure 20 compares the MDM, CDM and PBI (see Cen, Ostriker, & Peebles 1993) distributions of the misalignment angle θ between the actual velocity $\vec{v}$ and the predicted direction as a function of the limiting distance. The distribution of θ is broader in PBI, because the large-scale density fluctuations are larger. But MDM and CDM yield similar distributions.

Next, we examine in Figure 21 the distribution of results of estimating the mass density by setting the magnitude of the actual velocity equal to the magnitude of the sum in equation (10) truncated at distance R, and then solving for the apparent density parameter Ω_e. We see that it is necessary to study a very large volume in order to get a reliable estimate for Ω. At $R = 10h^{-1}\mathrm{Mpc}$ half of the observers

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in the MDM model would think that Ω was greater than 7! We also see that in MDM one would generally overestimate Ω on small scales significantly more than in the CDM or PBI cases. The sharp upturn of Ω_e as one goes to smaller scales is very interesting. The reason is that at present ($z = 0$) the relatively cold neutrinos still have a significant amount of thermal motion which makes the relatively shallow potential wells incapable of capturing them. Since we randomly (uniformly in volume) sample the space, we mainly sample the underdense regions (which occupy most of the space and where potential wells are shallow); in these regions, velocities (as well as densities and potentials) are in large part not induced by gravity, and therefore the apparent Ω_e does not represent the mass density on these small scales. These figures assume perfect data and complete sampling, which is clearly unatainable in practice. A more realistic set of assumptions would have further increased the dispersion. A similar set of conclusions with regard to the determination of H_0 was made by Turner, Cen & Ostriker (1992).

5. CONCLUSIONS

Our hydrodynamic simulations of the MDM scenario utilizing different cell sizes and box sizes to cover the dynamic ranges of interest are sufficiently accurate, we believe, to allow us to compute, with reasonable confidence, properties of the gas distribution on scales larger than 2.5 cell sizes and to compare with the standard CDM model computed with the identical numerical code. Our large PM simulation complements our hydro simulation on large-scales. Our results show that this MDM model, while normalized to COBE, appears to fare similarly as the standard CDM model with the same σ_8.

(1) Galaxy formation occurs somewhat later in MDM model than in the standard CDM model with the same σ_8. The galaxy formation fraction is about 0.01%

at $z = 3$ and peaks near $z \sim 0.3$ in the MDM model while in the standard CDM model the peak is around $z \sim 0.5$. At redshift $z = 4$ the MDM model has less galaxy formation by a factor of nearly one hundred than the CDM model having the same value of σ_8. Reducing Ω_b substantially could increase the power on small scales to up to 25% but we doubt that this would suffice to bridge the gap and it would produce other problems, since a lower baryon density reduces the cooling rate and thus inhibits galaxy formation.

(2) The soft X-ray radiation is far below and thus consistent with the observations by Wu $et\ al.$ (1991). But it can still make a non-trivial contribution to the observed soft X-ray background, approximately 12% of the residual (after taking into account of the half contribution from discrete sources) X-ray background. The Zeldovich-Sunyaev y parameters is computed to be $\bar{y} = (5.4 \pm 2.7) \times 10^{-7}$ with fluctuations $\bar{\delta y} = (6.0 \pm 3.0) \times 10^{-7}$ on arc minute scales, which are below current observational limits.

(3) With our scheme of identifying galaxy formation candidates at $z = 1$ we find that the final, computed bias of galaxy distribution over mass is ~ 4, a value which is larger than the assumed value ($b = 1.5$). But a more quantitative comparison between simulated galaxies at $z = 0$ with the observations awaits a simulation where galaxy formation is treated to be irreversible (like the one for CDM model of Cen & Ostriker 1993b,c). The two-point correlation function of galaxies has approximately the correct slope given our crude scheme of tagging galaxies.

(4) Using physical criteria for the formation of galaxies from cooling gas, we find that approximately the correct total mass density of baryons collapses to galaxies and that these have approximately the correct mass spectrum. Specifically, a reasonable fit to the observed Schecter luminosity function is obtained if $M_b/L_B = 4$ to give $M_{bar}^* = 5 \times 10^{10} M_\odot$ and $M_{tot}^* = 5 \times 10^{12} M_\odot$. Thermal en-

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ergy prevents the smallest scales from being most unstable with the result that the mass-weighted mass function is expected to decline for galaxy masses (in baryons) less than $2 \times 10^9 M_\odot$.

(5) The small-scale ($\Delta r = 1 h^{-1}$Mpc) velocity dispersion is 605 ± 8km/s, which might be reduced to 480 ± 6km/s by a reasonable physical bias, still somewhat in excess of the observed value. The large scale coherence of motion is almost identical to that in the CDM model.

(6) The cluster mass function is larger by a factor of about 4, and cluster-cluster 2-point correlation length slightly lower but marginally consistent with observations.

Overall, this model does not seem to be more successful than the standard CDM model with the same value of σ_8, but it is far better than standard CDM if both are normalized to COBE. Critical tests for this model check on whether or not it will provide enough nonlinear structure at early times. On the $10 - 100$kpc scale higher resolution simulations are required to see if galaxy formation can begin at an early enough epoch to satisfy Gunn-Peterson and other constraints. At the $0.1 - 10$Mpc scale higher resolution studies are needed to test if there are enough high central density clusters at moderate redshift in this picture to provide the gravitational lenses needed to make the observed luminous arcs.

While this proposed work is still to be done; it seems to us that no successful MDM model can or will be found. The reason is that observational constraints push the unknown, ratio $r_C \equiv \Omega_{CDM}/(\Omega_{CDM} + \Omega_{HDM})$ in opposite directions. We adopted $r_C = 0.7$, the same as other investigations (KHPR, DSS, TR). In order to produce early enough formation of quasars (Haehnelt 1993) or galaxies (this paper) and clusters of galaxies (this paper) a larger value of r_C should probably be adopted. But in order to match the cluster mass function or the small-scale velocity dispersion a smaller value of r_C is required. It is easy to see that variations

of the Hubble parameter h will not be able to overcome these difficulties. It may be that the observations are pushing us firmly towards a serious consideration of open $\Omega < 1$ models.

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FIGURE CAPTIONS

Fig. 1– Figure (1) shows the initial power spectra for cold dark matter component (thick solid line) and hot dark matter component (thick dashed line), and the final power spectra for cold dark matter component (thin solid line) and hot dark matter component (thin dashed line) for the four simulation boxes separately. The baryonic power spectrum is assumed to follow initially that of the cold dark matter component. The thick solid label shows the place in the spectrum at $8h^{-1}$Mpc, which we use to parameterize the amplitude of the spectrum.

Fig. 2– The upper panels of Figures (2a,b,c,d) show the evolution of mean volume-weighted (solid lines), and mass-weighted (dotted lines) temperatures as a function of redshift. Also shown is the corresponding mean proper peculiar kinetic energy density (dashed lines) in units of Kelvin. The lower panels of Figures (2a,b,c,d) show the density variances of baryonic matter (solid line) and dark matter (dotted line) [cf equation (1) for definitions]. Heavy lines show this work and light lines show CO92 run.

Fig. 3– Figures (3a,b,c,d) show the volume-weighted histograms of temperatures of cells at several epochs, at $z = 5$ (solid line), $z = 2$ (dotted line), $z = 1$ (solid line) and $z = 0$ (dotted line). The peaks at $T \sim 10^{4.5}$ K are mainly due to cooling by by hydrogen L_α lines. The temperature rises in the bigger boxes at late times are due to the ultimate breaking of long waves.

Fig. 4– Figures (4a,b,c,d) show the mass-weighted histograms of temperatures of cells at several epochs, at $z = 5$ (solid line), $z = 2$ (dotted line), $z = 1$ (solid line) and $z = 0$ (dotted line).

Fig. 5– Figures (5a,b,c,d) show some typical slices of baryonic matter density, dark matter density and baryonic matter temperature contour plots at

$z = 2$ and $z = 0$ for simulations with $L = 64, 16, 4h^{-1}$Mpc, respectively. All slices are 21 cells thick. The contour levels for densities are as following: $[1 + \sigma(\rho_b)]^{I/2}$ for $L = 64h^{-1}$Mpc and $L = 16h^{-1}$Mpc boxes, $[1 + \sigma(\rho_b)]^{I/4}$ for $L = 4h^{-1}$Mpc and $L = 1h^{-1}$Mpc boxes, with $\sigma(\rho_b)$ the density rms fluctuation in the baryonic matter. The contour levels for temperature are as following: $[1 + \sigma(T)]^{I/2}$ for $L = 64h^{-1}$Mpc and $L = 16h^{-1}$Mpc boxes, $[1 + \sigma(T)]^{I/4}$ for $L = 4h^{-1}$Mpc and $L = 1h^{-1}$Mpc boxes, with $\sigma(T)$ the rms temperature fluctuation in the baryonic matter; where I is positive integer, $\sigma(\rho_b)(z = 2) = 0.67, 1.12, 2.11, 3.38$, $\sigma(\rho_b)(z = 0) = 3.52, 8.45, 31.87, 20.76$, $\sigma(T)(z = 2) = 1.75, 2.92, 5.60, 6.92$, $\sigma(T)(z = 0) = 9.89, 16.83, 11.38, 5.05$, respectively, for $L = 64, 16, 4, 1h^{-1}$Mpc boxes.

Fig. 6– Figures (6a,b,c,d) show one typical slice of CDM density and HDM density at three $(z = 5, 2, 0)$ redshifts for simulations with $L = 64, 16, 4h^{-1}$Mpc, respectively. All slices are 21 cells thick.

Fig. 7– Figures (7a,b,c,d) show the (ρ, T) contour plots. The innermost contours represent the highest fraction of cells in terms of volume with these densities and temperatures at $z = 0$. Note that the smaller boxes have certain regions with high densities but relatively low temperatures while for bigger boxes this feature disappears. The contour levels are defined as follows: $10^{I/4}$, where I is positive integer, $I = 0$ corresponds to the outermost contour, and contours inside it have gradually increasing I.

Fig. 8– Figure (8) shows the collapsed galaxy fractions in four four simulations: thick, solid curve (MDM with $L = 64h^{-1}$Mpc), thick, dashed curve (MDM with $L = 16h^{-1}$Mpc), thin, solid curve (CDM with $L = 64h^{-1}$Mpc), thin, dashed curve (CDM with $L = 16h^{-1}$Mpc).

Fig. 9– Figure (9) shows the mean radiation at five epochs, at $z = 5$ (solid line),

$z = 3$ (dotted line), $z = 2$ (short, dashed line), $z = 1$ (long, dashed line) $z = 0.5$ (dotted-short-dashed line) and $z = 0$ (dotted-long-dashed line). The box in the middle shows the observational data by Wu *et al.* (1990).

Fig. 10– Figure (10) shows the galaxy-galaxy as well as dark matter particle-particle two-point correlation functions in the simulation of $L = 64h^{-1}$Mpc box at $z = 1$. Open and filled circles are the galaxy-galaxy two-point position correlation and the dark matter particle-particle two-point position correlation, respectively, The errorbars are one sigma Poisson fluctuations. The dotted line is which is the observational data (*cf.* Davis and Peebles 1983), $\xi(R) = (R/5h^{-1}\mathrm{Mpc})^{-1.8}$ scaled down by a factor of $1/2^2$ (This factor is the linear growth factor from $z = 1$ to $z = 0$ in this model). Note that the simulated galaxies are strongly biased with regard to the dark matter and the observed galaxies.

Fig. 11– Figure (11) shows the number density of groups as a functions of number of galaxies in the group identified in the $L = 64h^{-1}$Mpc box. This is related to the multiplicity function but a quantitative comparison with observations is not appropriate given our poor spatial resolution. The corresponding curve for the CDM model (CO92) is shown as the dashed histogram. We see that MDM model predicts a somewhat higher level of low end groups while at the high end the two models agree.

Fig. 12– Figure (12) shows the baryonic and total mass/multiplicity functions of collapsed objects. Open triangles, filled dots, filled triangles and filled squares are collapsed objects from four different boxes, with box sizes $L = (64, 16, 4)h^{-1}$Mpc, respectively. The upper panel of Figure (12) shows the baryonic mass/multiplicity function of collapsed objects, and the dashed line is a fitting formula [equation (4)], where M_{baryon} is in

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units of solar mass, the solid line is the derived Schechter function if mass to light ratio is 4.2 [equation (5)]. The lower panel of Figure (12) shows the total mass/multiplicity function of collapsed objects, and the dashed line is a fitting formula [equation (5)]. Data roughly fits the Schechter function if the baryon mass to blue light ratio is in the range 1.5 to 7.7.

Fig. 13– Figure (13) shows the initial (linearly scaled to $z = 0$) final power spectra of the PM simulation. The thick, solid line is the initial ($z = 20$) power spectrum of the cold dark matter component the MDM model. The thick, dashed line is the initial power spectrum of the hot dark matter component. The thin, solid line is the final ($z = 0$) power spectrum of the cold dark matter component. The thin, dashed line is the final power spectrum of the hot dark matter component. For the purpose of comparison also shown is the final (z=0) power spectrum for the COBE-normalized CDM model (thin dotted line).

Fig. 14– Figure (14a) shows the dark matter particle-particle two-point correlation functions from the large PM simulation ($L = 320h^{-1}$Mpc) at $z = 0$. The two dark matter components (CDM and HDM) are shown separately. The errorbars are one sigma Poisson fluctuations. The dotted line is which is the observational data (*cf.* Davis and Peebles 1983), $\xi(R) = (R/5h^{-1}\mathrm{Mpc})^{-1.8}$. Figure (14b) shows those at $z = 2$.

Fig. 15– Figure (15a) shows the computed cluster mass functions for MDM model as well as that from observations (Bahcall & Cen 1993). We believe that the discrepancy is significant. Also shown is that for the COBE-normalized CDM model. Figure (15b) shows the computed cluster mass functions for MDM model at three different epochs ($z = 0, 0.3, 1$). Note that the masses for the clusters are within $1.5h^{-1}$Mpc (metric not co-

moving).

Fig. 16– The two point correlation function of Abell $R \geq 1$ clusters in real space, with mean separation of $55h^{-1}$Mpc, from our simulated Abell clusters and from observations (Bahcall 1988) (the computed correlations on scales ▪ $r \leq 5h^{-1}$Mpc is probably underestimated due to our limited numerical resolution of the cluster identification scheme). We believe that the discrepancy is significant. Also shown is that for the COBE-normalized CDM model.

Fig. 17– A measure of cluster merging. The figure compares the distribution in the fractional mass change $\Delta M/M$ (eq. [11]) from redshifts $z = 0.3$ and $z = 1$ to the present, for clusters with number density $9.4 \times 10^{-6} h^3$Mpc^{-3}, in the MDM and CDM models.

Fig. 18– Figure (18a) shows the one-dimensional mass-weighted scalar relative velocity dispersion [see equation (8)] v_{1d} as a function of three-dimensional separation r. Also shown is the data from Davis and Peebles (1983) (which might reasonably corrected upwards by a factor of 1.25 to allow for velocity bias on this scale). Figure (18b) shows the 1-d velocity dispersion at $1h^{-1}$Mpc separation in 100 randomly selected $25h^{-1}$Mpc subboxes as a function of mass overdensity of the subboxes relative to the global mean for two case: the open circles with and filled dots without velocity pairs (> 1000km/s). Also shown are the value from KHPR (thin horizontal arrow at the left axis) and our computed value (thick horizontal arrow). The dashed histogram indicates the distribution (shown by the right vertical axis) of the subboxes as a function of their overdensities. Note that the average pairwise velocity dispersion (the open circles weighted by the dashed histogram) is not the same as indicated by the

thick solid arrow since the former is a volume-sampling and the latter is a particle-sampling.

Fig. 19– The mass-weighted scalar velocity autocorrelation function (eq. [9]) as a function of separation. Note that the velocity coherence length is about $110h^{-1}$Mpc, significantly larger than in the CDM model.

Fig. 20– Frequency distribution of the angle θ between the actual peculiar velocity $\vec{v}$ and the predicted direction as a function of the limiting distance, for MDM, CDM and PBI models.

Fig. 21– Distribution of the effective density parameter (eq. [10]) as a function of the limiting distance R for MDM, CDM and PBI. Note both the large dispersion and the systematic tendency to overestimate Ω as compared to the true values (indicated by arrows).

Table 1a. Summary of the volume weighted fractions in the $L = 64h^{-1}$Mpc box model

Redshift	10	5	3	2	1	0
(1) "Virialized Gas"	0.	0.	0.000000	0.000000	0.000004	0.000009
(2) "Galaxies"	0.	0.	0.000005	0.000062	0.000851	0.000456
(3) "Hot IGM"	0.	0.	0.000025	0.000217	0.004745	0.063292
(4) "Voids"	1.	1.	0.999969	0.999721	0.994267	0.936243

Table 1b. Summary of the mass weighted fractions in the $L = 64h^{-1}$Mpc box model

Redshift	10	5	3	2	1	0
(1) "Virialized Gas"	0.	0.	0.000000	0.000000	0.000049	0.000064
(2) "Galaxies"	0.	0.	0.000057	0.000799	0.012183	0.005024
(3) "Hot IGM"	0.	0.	0.000089	0.001105	0.034414	0.402218
(4) "Voids"	1.	1.	0.999854	0.998096	0.953032	0.585602

Table 2a. Summary of the volume weighted fractions in the $L = 16h^{-1}$Mpc box model

Redshift	10	5	3	2	1	0
(1) "Virialized Gas"	0.	0.000001	0.000008	0.000083	0.000301	0.000713
(2) "Galaxies"	0.	0.000007	0.000044	0.000354	0.001347	0.001333
(3) "Hot IGM"	0.	0.000000	0.000001	0.000007	0.000179	0.008122
(4) "Voids"	1.	0.999992	0.999947	0.999556	0.998173	0.989832

Table 2b. Summary of the mass weighted fractions in the $L = 16h^{-1}$Mpc box model

Redshift	10	5	3	2	1	0
(1) "Virialized Gas"	0.	0.000016	0.000101	0.001141	0.004796	0.014527
(2) "Galaxies"	0.	0.000099	0.000676	0.006354	0.030025	0.059315
(3) "Hot IGM"	0.	0.000000	0.000005	0.000118	0.010184	0.151940
(4) "Voids"	1.	0.999885	0.999217	0.992387	0.955071	0.774229

Table 3a. Summary of the volume weighted fractions in the $L = 4h^{-1}$Mpc box model

Redsift	10	5	3	2	1	0
(1) "Virialized Gas"	0.	0.000025	0.000011	0.000010	0.000025	0.000018
(2) "Galaxies"	0.	0.000000	0.000006	0.000021	0.000056	0.000025
(3) "Hot IGM"	0.	0.000000	0.000000	0.000000	0.000001	0.001876
(4) "Voids"	1.	0.999995	0.999983	0.999969	0.999922	0.998081

Table 3b. Summary of the mass weighted fractions in the $L = 4h^{-1}$Mpc box model

Redshift	10	5	3	2	1	0
(1) "Virialized Gas"	0.	0.000060	0.000163	0.000160	0.001175	0.003796
(2) "Galaxies"	0.	0.000000	0.000506	0.003213	0.013870	0.034122
(3) "Hot IGM"	0.	0.000000	0.000000	0.000000	0.000007	0.012791
(4) "Voids"	1.	0.999940	0.999331	0.996627	0.984948	0.949294

Table 4. Mass fraction and average mass of collapsed objects in the four models

L (h^{-1}Mpc)	64	16	4
$< m >$	$5.9 \times 10^{10} M_{\odot}$	$2.5 \times 10^9 M_{\odot}$	$1.3 \times 10^9 M_{\odot}$
f(collasped)	0.012	0.030	0.014

Table 5. The mean Zeldovich-Sunyaev y parameter as a function of redshift

Redshift	5	3	2	1	0.5	0
$\bar{y}$	5.0×10^{-12}	1.5×10^{-10}	6.0×10^{-10}	1.1×10^{-8}	2.7×10^{-8}	2.7×10^{-7}